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Recession and Recovery of the Pandemic

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Abstract

We develop an SIR-macroeconomic model with virus detection and inequality to study their implications for economic and health consequences during a pandemic crisis. We find a two-way relationship between the pandemic recession and inequality that exacerbate each other although such a vicious circle could be broken by accurate and extensive testing. This mitigation effect can be improved given complementary arrangements such as social protection. The extensive virus detection could not only be a better alternative intervention to lock-down to break the "life-or-economy" trade-off, but also prevent the economy to be permanently damaged if there is reinfection.

JEL classification: E1, H0, I1

Keywords: COVID-19, SIR-macro model, testing, inequality

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1 Introduction

The COVID-19 pandemic crisis generated far-reaching impacts on both public health and the economy. One year after the outbreak of the pandemic, more than 520 million people have been infected with millions of people dead. Moreover, the economic losses are unprecedently severe with many economies suffering their largest slump in economic growth since World War Two. Moreover, the adverse effects of the COVID-19 pandemic show a heterogeneous pattern that depends on the financial vulnerability of households (see Goldin & Muggah (2020) among others) with the poor tending to be more exposed to the pandemic than the rich.

Since the COVID-19 pandemic crisis is not an economic crisis alone, analysing the impacts of the pandemic requires a unified framework combining both epidemic dynamics and economic decisions. An essential question is what factors determine the recession and recovery from the COVID-19 pandemic? The goal in this paper is to understand the question by focusing on the interaction between an economic factor (inequality) and pharmaceutical interventions[1] (testing and quarantine). Moreover, we seek to understand the role of virus detection in the dynamics of the pandemic crisis. This effect was not well-recognized, especially in the early outbreak of the pandemic.

To facilitate this, we build a Susceptible-Infectious-Recovered-macro (SIR-macro) model to analyse the recession and recovery of the pandemic crisis. Consistent with other modelling of feedbacks between an epidemic and economic activities (Chari et al. 2021, Eichenbaum et al. 2021, Farboodi et al. 2021), our model features both epidemiological and economic blocks with endogenous feedbacks between the two parts.

In this paper, we augment this approach in several ways. In the macroeconomic parts, we classify household inequality by financial status. The wealthy not only earn a wage or salary but they also obtain dividends given their ownership of firms. On the contrary, the poor have to rely on a wage or salary for living and hence their income is more exposed than the wealthy, especially in quarantine. Such a classification of households is consistent with data showing that the majority of net wealth is held by the top half of households in the US (e.g. the wealthy in the model, see Figure 14 in Appendix B). Compared with SIR-macro models including more sophisticated wealth distributions, we provide a parsimonious way to approach inequality, the solution of which does not require nontrivial computational techniques (Debortoli & Galí 2018) thus there is no need to keep track of the distribution of wealth in the presence of pandemic evolution. Furthermore, for the epidemiological block, we incorporate virus testing, and for infected people, we distinguish between those detected and those undetected. The virus detection can identify undetected people who are infected and will enter quarantine. Compared with other macroeconomic models with testing, such as Aum et al. (2021) and Eichenbaum et al. (2022), we isolate the effect of testing and that of social protection. Accounting for this important difference provides valuable insights highlighting the ambiguous implications of virus detection for

[1] There are other important factors which are addressed in the literature (Baker et al. 2020, Carroll et al. 2020, Coibion et al. 2020, Eichenbaum et al. 2021, Elenev et al. 2020, Faria-e Castro 2021, Ganong et al. 2020). In terms of the economic sides, fiscal stimulus and loose monetary policies are adopted to support the survival of firms and households. For pharmaceutical factors, vaccination and treatment are important to end the widespread of the virus. Some non-pharmaceutical factors, such as social distancing and the use of face masks, are important to buy time for the arrival of pharmaceutical measures.

the poor in the early outbreak of the pandemic. Detection is useful since it helps to cut down the transmission path of the virus. However, the livelihood of the poor entering quarantine would significantly deteriorate in the absence of social protection. The combination of the epidemiological and economic aspects enables us to investigate the interaction of the virus detection and the inequality, to further shed light on the magnitude of the recession and shapes of recovery of the pandemic crisis.

Our model delivers important findings in several aspects. Firstly, we find that the pandemic crisis has heterogeneous effects on households with the poor being more affected due to their vulnerable income position. In turn, the presence of inequality exacerbates the pandemic recession and also leads to a sluggish recovery. Secondly, the adverse impacts of the pandemic crisis on both health and economic sides could be significantly mitigated by extensive testing at the aggregate level. The virus detection can reduce infection probability, which further encourages people to consume and work. Such an effect for the wealthy could be more apparent, compared with the poor. For the latter, they would enjoy benefits from the testing given complementary policies such as social protection policies which ensure their livelihood in quarantine.

Thirdly, testing and quarantine is an effective intervention tool to break the "life-or-economy" trade-off, induced by a lock-down. This finding implies that extensive testing could be an alternative tool to combat the pandemic crisis, and stresses the importance of medical preparedness in the early outbreak of the COVID-19 pandemic. And fourthly, we find that testing and quarantine is beneficial if reinfection is possible. The presence of reinfection is likely to undermine the economy permanently. Comparing the two types of households, the poor would be more affected by the loss of immunity or virus mutation. To deal with this situation, extensive testing could shield the economy from irreversible damage and prevent worsened inequality.

This paper is related to the rapidly growing literature on the implications of COVID-19, in particular, the interaction between the pandemic and the economy (Eichenbaum et al. 2021, Farboodi et al. 2021, Hall et al. 2020). In the literature, the epidemiological evolution is integrated into economic models to address the economic and health consequences simultaneously. Another strand of literature analyzes the dynamic of income and/or wealth inequality during the pandemic crisis (Adams-Prassl et al. 2020, Alon et al. 2020, Glover et al. 2020, Kaplan et al. 2020, Stantcheva 2022). Furthermore, since the pandemic crisis is not triggered by economic factors, some papers investigate the driving factors of the pandemic recession (Baqaee & Farhi 2020, Brinca et al. 2020, Guerrieri et al. 2020). In terms of policy interventions, the pandemic crisis has also spurred the evaluation of the effects of non-economic policies, such as pharmaceutical and non-pharmaceutical policies, on fighting the pandemic crisis (Acemoglu et al. 2020, Alvarez et al. 2020, Berger et al. 2020, Brotherhood et al. 2020, Chari et al. 2021, Eichenbaum et al. 2022, Krueger et al. 2022).

We contribute to the literature by developing a simple SIR-macro model with virus detection and inequality. Our results provide implications for the pandemic recession, and address some potential challenges for the recovery. In particular, we show that the virus detection and quarantine is an important element determining the recovery dynamics. An efficient and high level of detection rate could lead to a V-shaped recovery while an inaccurate and low level of detection rate could relatively delay the recovery and lead it to be U-shaped. The recovery speed could be further delayed due to the presence of income inequality. An L-shaped recovery is likely when reinfection becomes possible and there is no sufficient detection to deal with this situation.

The rest of the paper is organized as follows. Section 2 provides some motivational evidence followed by Section 3 that outlines the model with virus detection and inequality. Section 4 describes our parameter calibrations. In section 5, we present our quantitative analysis. Section 6 concludes with comments.

2 Motivational Evidence

In this section, we provide empirical evidence to support the model's mechanism, particularly focusing on the relationship between economic growth and inequality or virus detection in the COVID-19 pandemic period.

2.1 Inequality and growth

In this subsection, we examine the role of the pandemic in the inequality-growth relationship. To this end, we first apply cross-sectional data in 2020 based on the World Development Indicators[2] to explore the inequality-growth relationship after the pandemic following the model specification below.[3]

$$Growth_i = \alpha_0 + \alpha_1 Gini_i + \alpha_2 X_i + \epsilon_i \tag{1}$$

where $Growth_i$ denotes economic growth for country i, measured by either GDP growth or GDP per capita growth rate, $Gini_i$ represents the inequality. X_i is a set of control variables including population, CPI, lagged GDP growth rate, lagged health expenditure, government spending, household consumption and employment. ϵ_i denotes regression errors.

Second, the inequality-growth relationship is further investigated by employing a fixed-effect (FE) panel data model from 2001 to 2020 in order to compare the inequality-growth relationship in general and that specifically in the COVID-19 pandemic period. We therefore treat 2020 as the pandemic year and interact it with the Gini coefficient and estimate the following specification (2).

$$Growth_{it} = \alpha_0 + \alpha_1 Gini_{it} + \alpha_2 Pandemic_{it} + \alpha_3 Gini_{it} * Pandemic_{it} + \alpha_4 X_{it} + \alpha_5 Y_i + \alpha_6 Z_t + \epsilon_{it} \tag{2}$$

where $Pandemic_{it}$ is a dummy variable (=1 if in the year 2020), capturing the effect of the pandemic, X_{it} is the same control variable matrix as in Equation (1) and Y_i represents the country fixed effects and Z_t

[2]The 2020 cross-section sample includes 92 countries for which we could obtain data for the Gini coefficient.
[3]The selection of control variables follows the classic literature in economic growth, e.g., Barro (1996). Detailed descriptions of the variables are included in Table 4 in Appendix B.

represent the year fixed effects[4]. Importantly, the estimate of coefficient α_3 captures the inequality-growth relationship in the pandemic period.

Table 1 reports the estimated relationships between inequality and economic growth. Columns (i) and (ii) show results based on (1), and columns (iii) and (iv) show results based on (2). The results show that in the cross-sectional model (1) the coefficient of *Gini* is negative and significant, suggesting a negative inequality-growth relationship during the pandemic. This finding is further confirmed by the panel data model (2) which shows that the estimated coefficient for the interaction term $Gini_{it} * Pandemic_{it}$ is negative and significant, suggesting that the growth rate is lower in the pandemic year for a country with a higher level of inequality. Interestingly, the estimated coefficient of *Gini* is positive and insignificant in the columns (iii) and (iv), suggesting that inequality is more likely to be a drag on growth in the pandemic rather than in normal times.

Table 1 Inequality-Growth relationship in the pandemic

Variables	2020 Cross section		2001-2020 panel	
	GDP growth (i)	GDP per capita growth (ii)	GDP growth (iii)	GDP per capita growth (iv)
Gini	-0.118*	-0.153***	0.043	0.0269
	(0.0615)	(0.0584)	(0.0266)	(0.0268)
*Gini*Pandemic*			-0.0719*	-0.0620#
			(0.0383)	(0.0384)
Pandemic			-5.290***	-5.332***
			(1.473)	(1.484)
pop	0.425		1.968*	
	(0.309)		(1.037)	
cpi	-0.00162	-0.00152	-0.00339***	-0.00343***
	(0.00206)	(0.00196)	(0.00101)	(0.00102)
l.gdpg	0.00722	-0.0122	0.110***	0.107***
	(0.276)	(0.261)	(0.0190)	(0.0192)
l.health_exp	-0.331	-0.933	-0.109	-0.116
	(0.322)	(0.302)	(0.132)	(0.132)
gov	-0.246**	-0.315**	0.0396	0.0458*
	(0.122)	(0.108)	(0.0248)	(0.0250)
con	-0.0453	-0.0492	-0.0915***	-0.0977***
	(0.0417)	(0.0396)	(0.0137)	(0.0138)
employ	0.094	0.0478	0.157***	0.0967***
	(0.0621)	(0.000592)	(0.0307)	(0.0301)
Obs	92	92	2477	2477
R^2	0.2320	0.2033	0.5146	0.4847

Note: robust standard errors are reported in parentheses; *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$, # $p = 0.1$. All variables are defined in Table 4 in Appendix B.

[4] Note that $Pandemic_{it}$ is a special term of yearly FE.

2.2 Testing and growth

In this subsection, we examine the test-growth relationship. As the pandemic spreads in short periods and testing policies changes rapidly, yearly data is insufficient and might be inappropriate. Given the relative data availability advantage of OECD countries, we focus on the quarterly panel data based on OECD countries to estimate the relationship between COVID-19 testing and economic growth. The testing data is from European Centre for Disease Prevention and Control (ECDC). We explore the testing-growth relationship after the pandemic using the following specification:

$$Growth_{it} = \alpha_0 + \alpha_1 Test_{it} + \alpha_2 X_{it} + \alpha_3 Y_i + \alpha_4 Z_t + \epsilon_{it} \tag{3}$$

where $Test_{it}$ represents COVID-19 testing rate per 100,000 people, and X_{it} is a set of control variables including population, CPI, lagged GDP growth rate, government spending, and household consumption.

Table 2 Testing-Growth relationship in the pandemic

Variables	Full (v)	2020Q1-Q2 (vi)	2020Q3-2021Q4 (vii)
Test	0.470*	0.335	0.432*
	(0.256)	(0.854)	(0.237)
pop	0.0128**	0.000891	0.0141***
	(0.0054)	(0.0165)	(0.00523)
cpi	0.114***	0.158	0.0926***
	(0.0278)	(0.118)	(0.0257)
con	-0.864	-4.559	-0.273
	(0.958)	(2.694)	(0.907)
gov	2.390**	4.509	2.169**
	(1.085)	(2.823)	(1.029)
inv	-1.376	-0.903	-1.712*
	(1.032)	(2.917)	(0.969)
l.gdpg	-0.342***	0.480	-0.424***
	(0.0716)	(0.431)	(0.0656)
l.gdp	-0.0275**	0.00593	-0.0295**
	(0.0120)	(0.0331)	(0.0114)
Obs	164	22	142
R^2	0.8129	0.7476	0.7009

Note: the dependent variable is GDP growth. Others are the same as above.

Table 2 presents the estimation results based on specification (3). The full sample results shown in column (v) show that the estimated *Test* coefficient is positive and significant at 10% level, suggesting a positive relationship between testing and growth. We further consider that many countries implemented job retention schemes after the outbreak of the pandemic. Effects of such schemes may affect the testing-growth relationship, which could be captured by the full sample regressions. Moreover, as Figure 15 in Appendix B suggests, potentially the relationships differed over the sample period if split after the first two quarters of

2020 given that the implementation of such schemes took time and were very limited in the first two quarters of 2020. Hence, we conduct sub-sample regressions, splitting the sample at 2020Q3 and the results are given in columns (vi) and (vii) of Table 2. This shows that although the estimated *Test* coefficient is positive over both periods it is only statistically significant in the later stage of the pandemic, implying that testing alone may not have been effective to combat the pandemic recession.

The motivational evidence in the sub-sections above provides empirical support to the main mechanism of our model detailed below. In particular, the empirical results are consistent with the model predictions that, a higher degree of inequality exacerbates the economic loss in the pandemic periods, which could be mitigated by extensive testing provided by other rescue schemes.

3 The Model

We build an SIR-Macro model with heterogeneous agents. There are two types of households with different equity holding: "wealthy" and "poor". The wealthy households are owners of firms and hence enjoy dividend payment as extra income. The poor households rely only on wages as income.

In terms of the epidemic block of the model, we incorporate testing of infected people in a conventional SIR model (Kermack & McKendrick 1927). By doing so, we distinguish between detected and undetected infectious people with the former entering quarantine and hence being no longer be infectious.

3.1 Firm

The representative monopolistic firm use labour N_t to produce output Y_t based on the following production function

$$Y_t = AN_t \tag{4}$$

where A is the productivity of labour. The profit π_t^f for the representative firm is

$$\pi_t^f = p_t Y_t - mc_t Y_t = p_t A N_t - w_t N_t \tag{5}$$

Optimal price setting implies that the price is equal to a mark-up λ times the marginal cost mc_t.

$$p_t = \lambda mc_t \tag{6}$$

where λ is the price mark-up. The marginal cost and the firm profit are

$$mc_t = \frac{w_t}{A} \tag{7}$$

$$\pi_t^f = (\lambda - 1) mc_t Y_t = (\lambda - 1) w_t N_t \tag{8}$$

3.2 Epidemic transition

We incorporate epidemiology dynamics that models the transition of the health status of households. The population can be divided into four categories: *susceptible* (people who have not yet been exposed to the disease), *infected* (people who contracted the disease), *recovered* (people who survived the disease and acquired immunity), and *deceased* (people who died from the disease). The fractions of people in these four groups are denoted by S_t, I_t, R_t and D_t, respectively. The number of newly infected people is denoted by T_t. Within the I_t category, we further distinguish between *detected* and *undetected* infections. The former refers to infected people who are also tested and diagnosed while the latter refers to infected people who are not tested and unaware if they are infected. Specifically, undetected people may be asymptomatic[5] or show mild symptoms which are hard to distinguish from other disease, such as seasonal flu.[6] We label these two sub-categories as I_t^d and I_t^u respectively.

Following Eichenbaum et al. (2021), suspected people can become infected through three ways: purchasing consumption goods, meeting at work, and random meeting with contagious people or materials.

The total number of newly infected people is given by:

$$T_t = \underbrace{\pi_1(S_t C_t^s)(I_t^u C_t^{iu})}_{\textit{due to consumption}} + \underbrace{\pi_2(S_t N_t^s)(I_t^u N_t^{iu})}_{\textit{due to working}} + \pi_3 S_t I_t^u \tag{9}$$

where π_1, π_2, and π_3 are parameters governing the magnitude of each source of infection. Comparing with Eichenbaum et al. (2020, 2021), we assume that only undetected people are infectious. detected people enter quarantine and hence they would not be infectious.

The evolution of each category of people are given by:

$$S_{t+1} = S_t - T_t \tag{10}$$

$$I_{t+1}^u = I_t^u + T_t - (\pi_r + \pi_d + \pi_u)I_t^u \tag{11}$$

$$I_{t+1}^d = I_t^d + \pi_u I_t^u - (\pi_r + \pi_d)I_t^d \tag{12}$$

$$I_t = I_t^d + I_t^u \tag{13}$$

$$R_{t+1} = R_t + \pi_r I_t \tag{14}$$

$$D_{t+1} = D_t + \pi_d I_t \tag{15}$$

$$Pop_{t+1} = Pop_t - \pi_d I_t \tag{16}$$

where π_r, π_u and π_d denote probability of recovery, detection and decease respectively. Note that the increase

[5] Long et al. (2020) find that asymptomatic patients may account 20% of infected people.

[6] In the early outbreak of the COVID-19, many infected people could not be tested.

of π_u may capture larger coverage of testing as in Eichenbaum et al. (2022) and more accurate testing.

3.3 Households

We classify households by health and income conditions. The potentially healthy status is defined in Section 3.2. In terms of the income status, a fraction χ of households are wealthy while the remaining $1-\chi$ are poor. Both types of households enjoy wage incomes but only wealthy people are owner of firms and hence obtain firm profits. The poor may be also interpreted as the working classes and the wealthy people as entrepreneurs.

Next, we describe the optimization problems for each type of agent. The upper index $i(i{=}s,iu,r)$ denotes the health status and $j(j{=}w,p)$ denotes the income status. The utility function (Eichenbaum et al. 2021) and the budget constraint for a type-i,j person is

$$u(c_t^{i,j}, n_t^{i,j}) = lnc_t^{i,j} - \frac{\theta}{2}(n_t^{i,j})^2 \tag{17}$$

$$c_t^{i,j} = w_t n_t^{i,j} + \mathbb{1}\pi_t^f \tag{18}$$

where $c_t^{i,j}$ and $n_t^{i,j}$ denote consumption and hours worked respectively. $\mathbb{1}$ is an indicator variable equal to one if the household is wealthy.

Susceptible people The lifetime utility of representative suspected people is

$$U_t^{s,j} = u(c_t^{s,j}, n_t^{s,j}) + \beta[(1-\tau_t)U_{t+1}^{s,j} + \tau_t U_{t+1}^{iu,j}] \tag{19}$$

where τ_t is the infection probability

$$\tau_t = \pi_1 c_t^s(I_t^u C_t^{iu}) + \pi_2 n_t^s(I_t^u N_t^{iu}) + \pi_3 I_t^u \tag{20}$$

Optimization yields

$$\frac{1}{c_t^{s,j}} = \lambda_t^{s,j} + \beta\pi_1 I_t^u C_t^{iu}(U_{t+1}^{s,j} - U_{t+1}^{iu,j}), \qquad j = w, p \tag{21}$$

$$\theta n_t^{s,p} = \lambda_t^{s,p} w_t - \beta\pi_2 I_t^u N_t^{iu}(U_{t+1}^{s,p} - U_{t+1}^{iu,p}) \tag{22}$$

$$\theta n_t^{s,w} = \lambda_t^{s,w} A\Theta_t - \beta\pi_2 I_t^u N_t^{iu}(U_{t+1}^{s,w} - U_{t+1}^{iu,w}) \tag{23}$$

where λ_t^s is the Lagrange multiplier associated with the budget constraint (18). $\Theta_t = \dfrac{S_t + I_t^u + R_t}{S_t + I_t + R_t}$ is an adjustment factor, capturing that wealthy people in the infected detected category earn dividend payment but do not work to produce output.

Infected undetected people The lifetime utility of infected undetected people is

$$U_t^{iu,j} = u(c_t^{iu,j}, n_t^{iu,j}) + \beta[(1 - \pi_u - \pi_r - \pi_d)U_{t+1}^{iu,j} + \pi_u U_{t+1}^{id,j} + \pi_r U_{t+1}^{r,j}] \tag{24}$$

Optimization yields

$$\frac{1}{c_t^{iu,j}} = \lambda_t^{iu,j}, \qquad j = w, p \tag{25}$$

$$\theta n_t^{iu,p} = \lambda_t^{iu,p} w_t \tag{26}$$

$$\theta n_t^{iu,w} = \lambda_t^{iu,w} A\Theta_t \tag{27}$$

Infected detected people The lifetime utility of infected detected people is

$$U_t^{id,j} = u(c_t^{id,j}, n_t^{id,j}) + \beta[(1 - \pi_r - \pi_d)U_{t+1}^{id,j} + \pi_r U_{t+1}^{r,j}] \tag{28}$$

Detected people enter quarantine immediately after detection and they would stop working (Eichenbaum et al. 2022). Hence their wage income becomes zero. In this case, the rich consume profit income while the consumption of the poor becomes zero. This might be an extreme assumption but it allows us to highlight different degrees of vulnerability of households to the pandemic crisis. Moreover, this assumption is consistent with the data used in our calibration below, which shows that the share of wealth held by the bottom half of households (i.e., the poor in the model) is very small and they rely on wage income for living. In Section 5.3, we relax this assumption and allow households to receive social protection. Comparatively, the model in Eichenbaum et al. (2022) implies that detected people receive consumption through government transfers. Our model separates the wage income from government transfers thus allowing us to focus on the effect of detection alone in our benchmark model.

Recovered people The lifetime utility of is recovered people[7] is

$$U_t^{r,j} = u(c_t^{r,j}, n_t^{r,j}) + \beta U_{t+1}^{r,j} \tag{29}$$

Optimization yields

$$\frac{1}{c_t^{r,j}} = \lambda_t^{r,j}, \qquad j = w, p \tag{30}$$

$$\theta n_t^{r,p} = \lambda_t^{r,p} w_t \tag{31}$$

$$\theta n_t^{r,w} = \lambda_t^{r,w} A\Theta_t \tag{32}$$

[7] The recovery probability may also depend on the financial condition of household. To keep traceability of the model, we do not include this type of heterogeneity.

3.4 Equilibrium

In equilibrium, each household optimizes their decisions and both the goods and the labour market clear.

$$S_t C_t^s + I_t^u C_t^{iu} + I_t^d C_t^{id} + R_t C_t^r = A N_t \tag{33}$$

$$S_t N_t^s + I_t^u N_t^{iu} + R_t N_t^r = N_t \tag{34}$$

$$C_t^i = \chi c_t^{i,w} + (1-\chi) c_t^{i,r}, \qquad i = s, iu, id, r \tag{35}$$

$$N_t^i = \chi n_t^{i,w} + (1-\chi) n_t^{i,r}, \qquad i = s, iu, id, r \tag{36}$$

4 Calibration

Table 3 reports the calibrated parameter values used for the quantitative analysis. Each period corresponds to a week.

Table 3 Calibrated parameters

Parameters	Description	Value
π_r	recovery prob	0.3869
π_d	decease prob	0.0019
P_0	initial population	1
t_0	initial infected people	0.001
π_u	detection prob	[0,0.6]
β	discount factor	0.9992
λ	price mark-up	1.35
H	ss labour hour	28
A	ss productivity	39.8352
χ	ss share of wealthy people	0.5

In terms of the parameter values related to pandemic evolution, we closely follow Eichenbaum et al. (2021) except for the detection rate π_u which is not present in the literature. As suggested by Atkeson (2020), it takes 18 days to recover or die from infection. Hence, we set $\pi_r + \pi_d$=7/18. The mortality rate is set as 0.5%, falling in the range (0.4%-0.7%) suggested by the US data. This implies π_d is 0.005∗7/18. Following the estimations from Eichenbaum et al. (2021), the infection parameters π_1, π_2, π_3 are calibrated as $7.8408 * 10^{-8}$, $1.2442 * 10^{-4}$, and 0.3902, respectively. In Eichenbaum et al. (2021), they estimate virus transmission related to consumption and working, and further match π_1, π_2, π_3 with these estimation results.

The discount factor is calibrated as $0.96^{1/52}$ on a weekly basis. The steady state labour hour H and productivity A are set as 28 and 39.8352 respectively to match the weekly working hour and income data from the U.S. Bureau of Economic Analysis. The share of wealthy household χ is set as 0.5, consistent with the fact that more than 98% of net wealth is held by top 50% of wealth percentiles[8]. Finally, the price mark-up λ is set as 1.35.

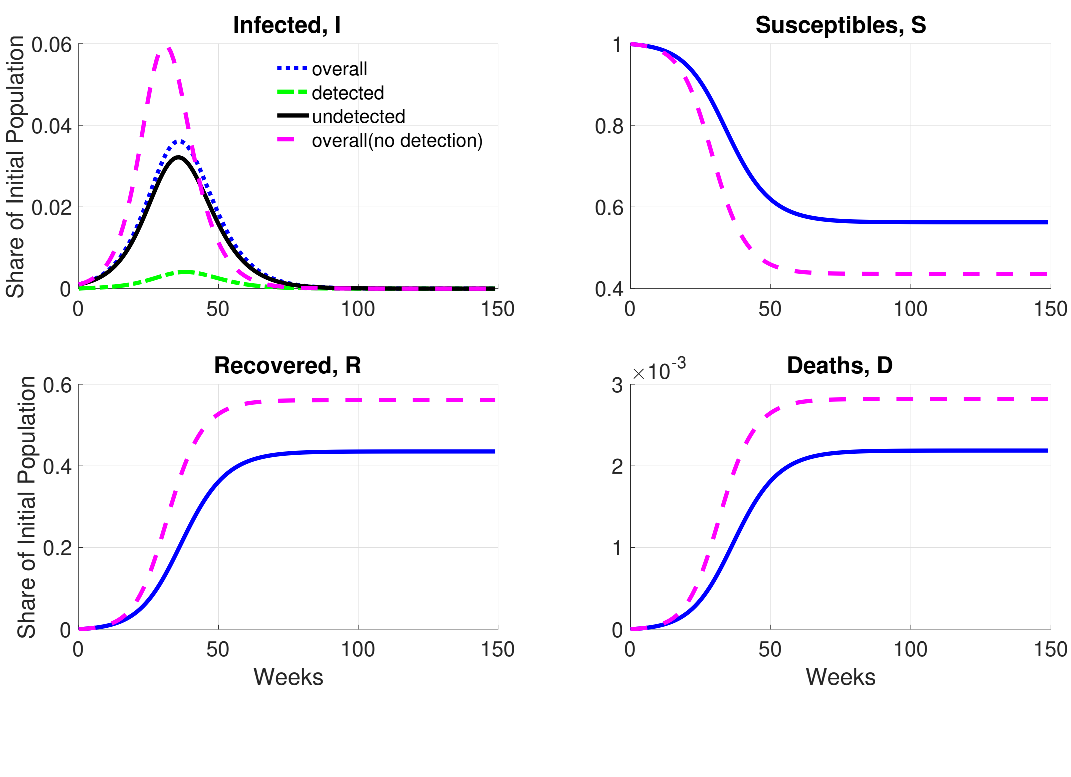
5 SIR-Macro Model Results

We start the analysis for the benchmark case where household incomes are equal. In such a case, the focus is on the implications of the detection on both the health and economic aspects. Then we present results based on the extended model with heterogeneous income and compare with the benchmark results. Through the comparison we emphasize the roles of inequality in the pandemic recession and how the inequality is interacted with virus detection.

5.1 Implications of the detection

Figure 1 and 2 respectively display the population dynamics and economic impacts following the outbreak of the pandemic. For the illustration purpose, we set the detection rate as 5%. We use a relatively low detection rate with the consideration that testing could be difficult and inaccurate at the beginning of the pandemic outbreak. In spite of this, a more comprehensive investigation is presented later.

Figure 1 The evolution of the epidemic



Compared with the case without detection as in Eichenbaum et al. (2021), there is a decrease in the

[8]Source: The Distributional Financial Accounts, https://www.federalreserve.gov/releases/z1/dataviz/dfa/index.html

amount of people in the infected and death categories. This finding is not surprising since the detected people would enter quarantine and hence the transmission probability would be cut down.

Turning to the economic sides, Figure 2 shows that the presence of detection could also mitigate the magnitude of the pandemic recession. In such a case, the decline of aggregate consumption and labour hours are dampened (see blue lines in Figure 2). Comparing the three categories of households, the recovered people are the least affected, followed by susceptible, while infected people are the most affected. The latter result is due to the reason that quarantined people (after detection) could not work and hence their consumption would be also limited.

Figure 2 Impacts on consumption and hours

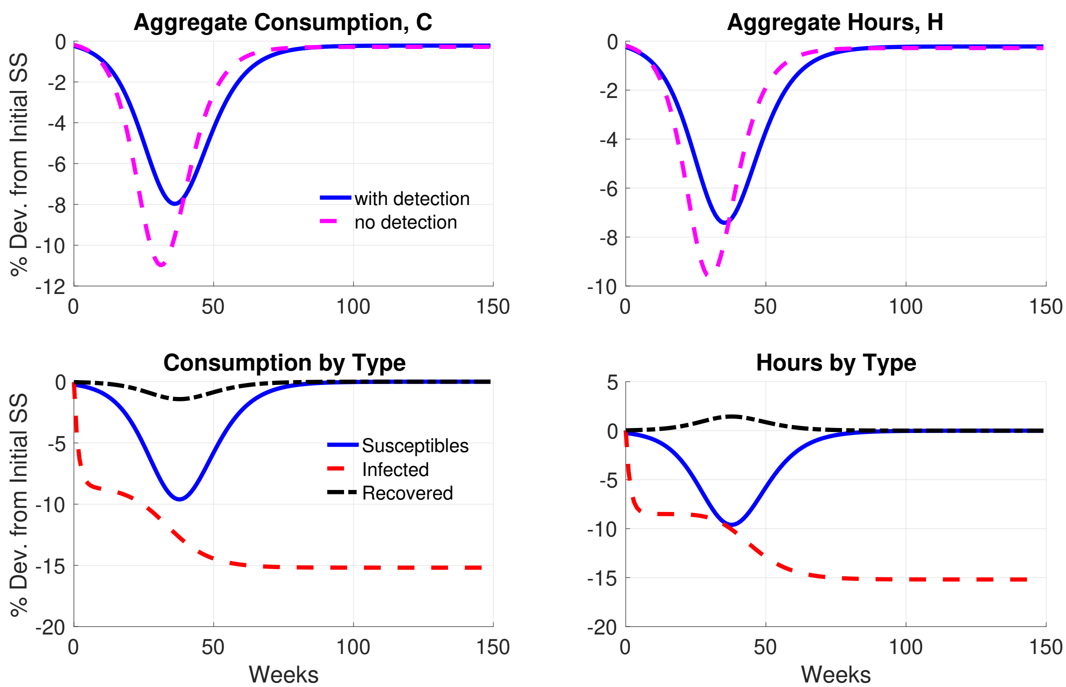
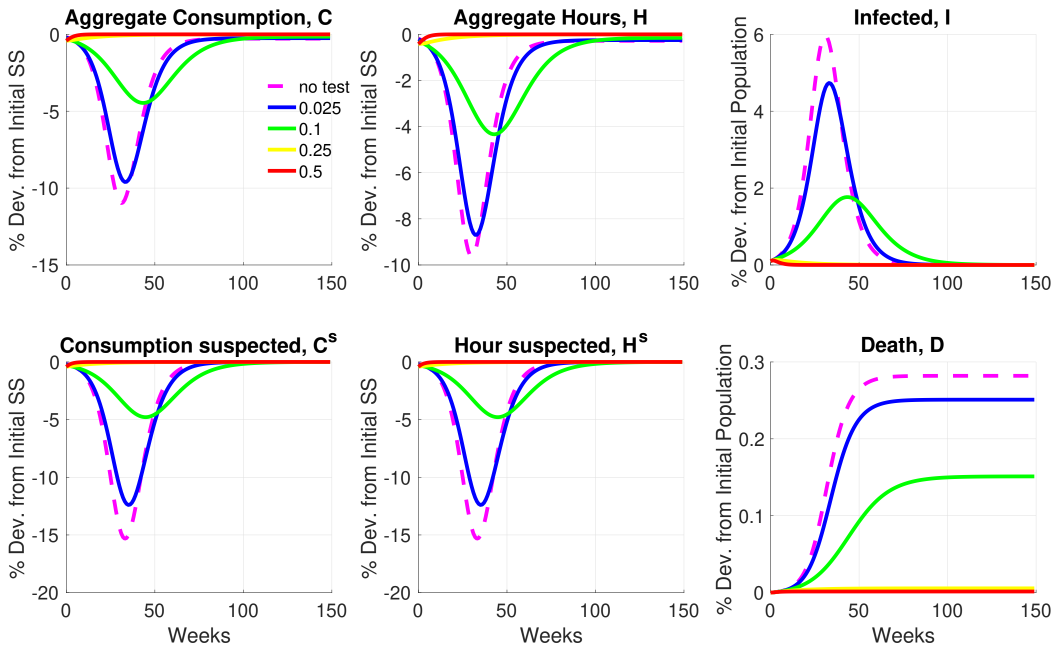


Figure 3 plots impacts of the pandemic with different detection rates on consumption (left panels), hours (middle panels), and health outcomes (right panels). Starting from the non-testing case, both the economy and household health suffer the most from the pandemic crisis. When the detection rate increases, the magnitude of the recession, infection, and mortality gradually dampens. For example, the largest loss of aggregate consumption is about 5% at 10% of detection rate (green line), halved as in the case with 2.5% of detection rate (green line). If the detection rate is even higher, say 50%, the impacts of the pandemic on both economic and health sides could become limited. Moreover, at high detection rates, the red and yellow lines in Figure 3 show that the evolution of the economy could exhibit different patterns compared with the low detection case. In the former cases, the economy rebounds quickly as the transmission path of the pandemic is quickly cut down – a V-shaped recovery. For the latter cases, low detection rates lead

Figure 3 Impacts of different detection rates



to sluggish containment of the pandemic and the influence on the economy is prolonged. Consequently, not only the magnitude of the recession is more sizeable, but also the recovery is relatively slow, leading to an U-shaped recovery.

5.2 The presence of inequality

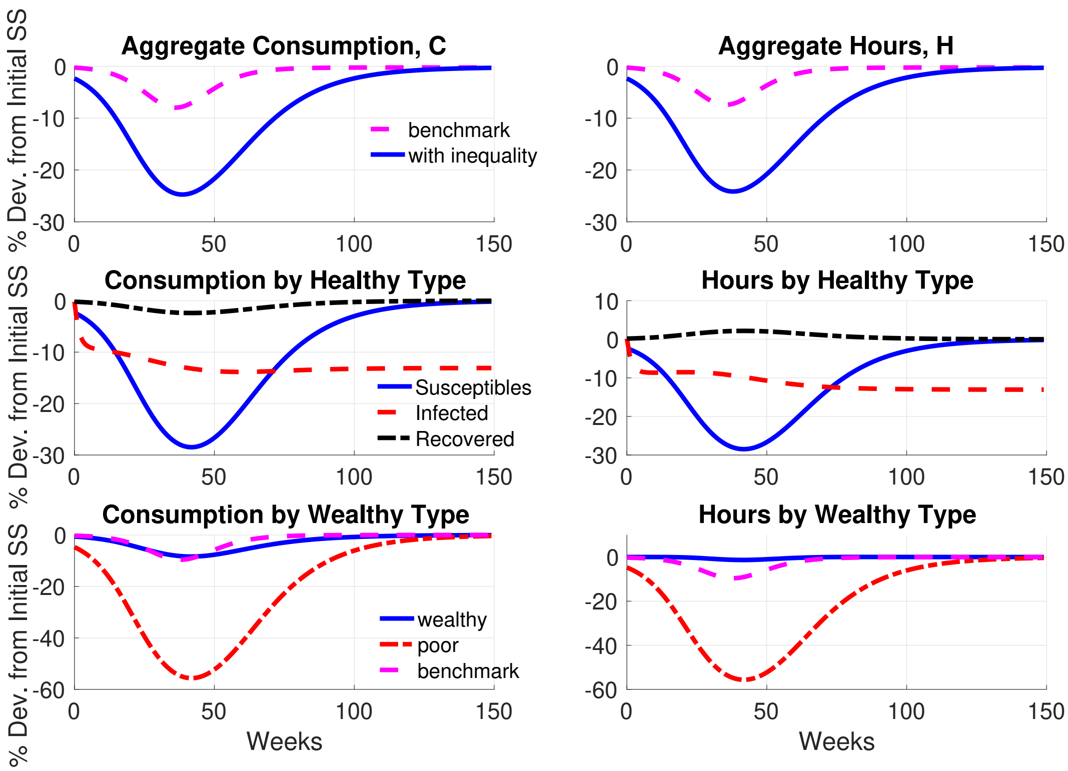
In this section, we relax the homogeneous income assumption and allow a fraction of households, the wealthy, to obtain all firm dividends. Figure 4 reports the impacts of the pandemic on the economy.

The upper panels of Figure 4 compare the economic dynamics at the aggregate level between the benchmark and the inequality cases. This shows significant differences in the response of aggregate consumption and hours. The presence of income heterogeneity significantly exacerbates the recession, leading to a larger magnitude of loss and slow recovery.

Moving attention to the middle panels of Figure 4, they show that the susceptible category is the most affected due to the presence of the inequality. Compared with Figure 2, the largest loss of suspected households could be near 30%, four times larger as in the benchmark case. While we are cautious in interpreting the quantitative results, the sizeable difference indeed suggests a significant role of the inequality in exacerbating the recession.

The lower panels of Figure 4 show consumption and hours for households classified by different wealthy levels. The impacts on the rich are similar to the benchmark level, both of which are comparatively lower

Figure 4 Impacts on consumption and hours: with inequality

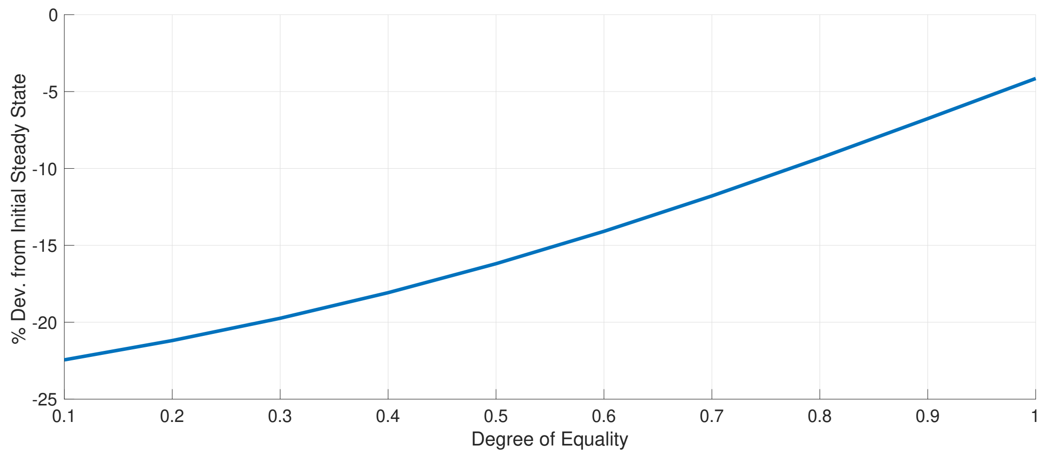


than those on the poor. Since the poor only have one source of income in the model, it is not surprising that they are vulnerable to the pandemic crisis.

To further explore the implication of income heterogeneity, we investigate a relationship between inequality and the magnitude of the recession. Figure 5 plots the relationship between (in-)equality and 1-year loss of aggregate consumption. A larger (smaller) value on the horizontal axis denotes a larger degree of (in-)equality and less (more) significant income heterogeneity. Specifically, the figure shows a positive relationship between the magnitude of the recession and the degree of inequality. This result further corroborates the finding that the presence of inequality exacerbates the pandemic recession. Moreover, these results are consistent with the motivational empirical evidence presented in Section 2 (see Table 1) regarding the inequality-growth relationship in the pandemic crisis.

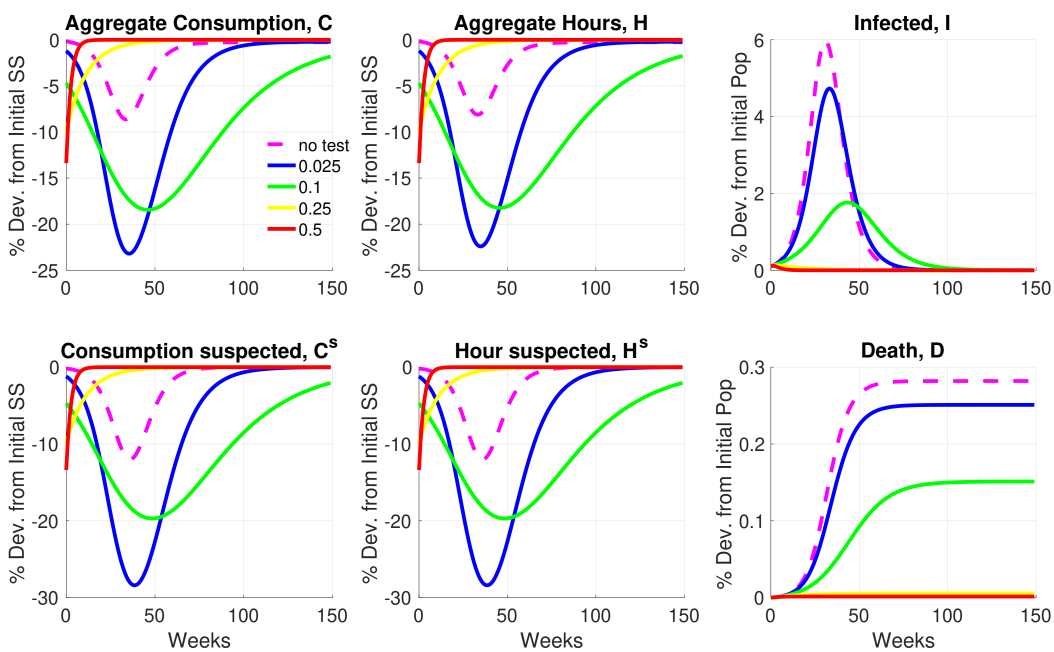
After establishing the implications of the inequality, we further investigate its interaction with detection to further shed light on the pandemic crisis. To this end, the same experiment as in Figure 3 is performed but based on the heterogeneous income model. The results are shown in Figure 6. Contrast to the economic impacts as in Figure 3, the magnitude of the recession does not show a monotonic decreasing relationship with detection rates when inequality is present. Instead, the relationship is found to be nonlinear. For relatively low detection rates (e.g., 2.5% and 5%), the magnitude of the recession increases with detection. While the

Figure 5 Implications of inequality for consumption loss



Note: This figure shows the relationship between 1-year aggregate consumption loss and degree of inequality.

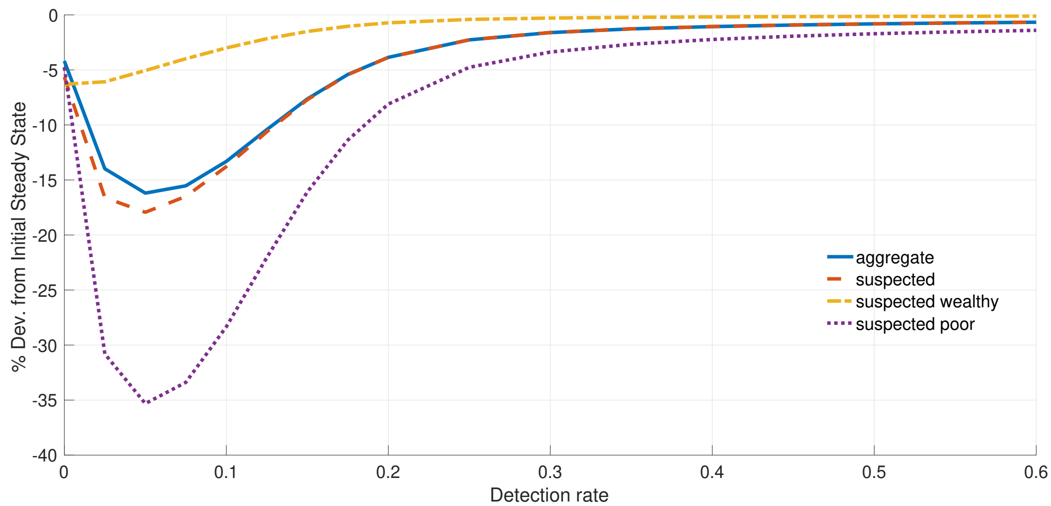
Figure 6 Impacts of different detection rates: with inequality



relationship turns to be decreasing when detection rates become high (e.g., 25% and 50%). These findings imply a U-shaped relationship between detection and magnitude of recession. Such a finding is confirmed in Figure 7.

Essentially, Figure 7 shows U-shaped relationships between the detection rate and 1-year averaged consumption loss at the aggregate level, for the susceptible category and poor people. On the contrary, the relationship for wealthy people is negative. The relationship at the aggregate level (blue curve) is driven by

Figure 7 Consumption loss and detection



Note: This figure shows relationships between 1-year averaged consumption loss and detection for aggregate case, suspected people, wealthy people and poor people.

the relationship between detection and poor people. As assumed by the model, the poor will lose all income after detected as infection. On one hand, increasing the test rate would reduce transmission probability, which encourages working and consumption, leading to less significant recession. On the other hand, higher test rates add pressure for the poor in the fear of being detected and losing all incomes. Hence, they also try to avoid virus transmission by cutting down consumption and working. To see the second mechanism, we borrow the equilibrium conditions of poor people (37) and (38) for explanations.

$$U_t^{iu,p} = u(c_t^{iu,p}, n_t^{iu,p}) + \beta[(1 - \pi_u - \pi_r - \pi_d)U_{t+1}^{iu,p} + \pi_r U_{t+1}^{r,p}] \tag{37}$$

$$\frac{1}{c_t^{s,p}} = \lambda_t^{s,p} + \beta\pi_1 I_t^u C_t^{iu}(U_{t+1}^{s,p} - U_{t+1}^{iu,p}) \tag{38}$$

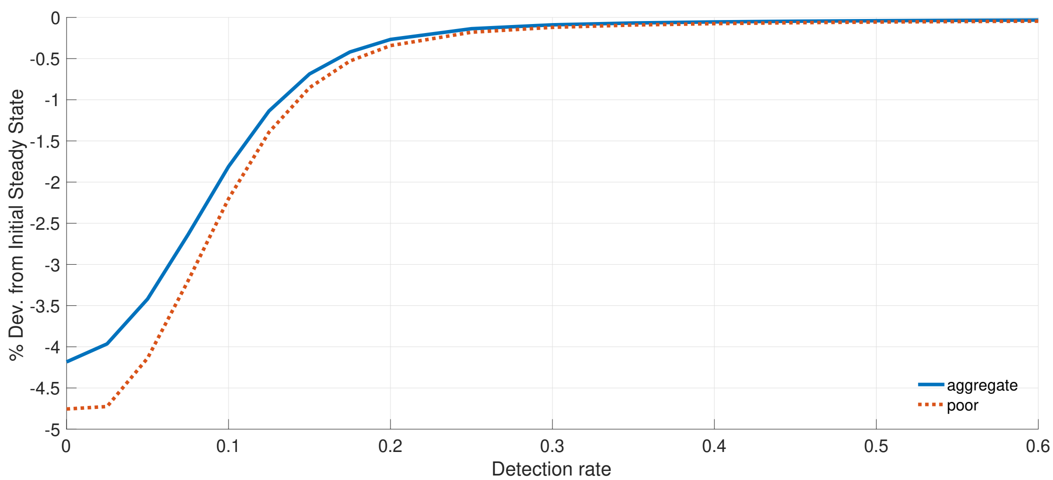
According to eq (37), the increase of detection rate π_u decreases lifetime utility for the poor if they are infected. Given others as constant, the utility gap $U_{t+1}^{s,p} - U_{t+1}^{iu,p}$ would be broadened. With this consideration, the suspected poor people could reduce consumption, as implied by eq (38).

The two counteracting forces play quantitatively different roles at different detection levels. Our results imply that the former force would be relatively more powerful when the detection rate becomes high. In terms of wealthy people, they have an alternative source of income. Even under quarantine, they can still earn dividends owing to their firm ownership. Hence, the role played by the second mechanism may not overweight the first one; the wealthy benefit more from detection than the poor for the economic side. Despite the asymmetric economic impacts of detection, its effects on health outcomes are positive for both the 2 groups of people. By visual check, we find the health outcomes in Figure 6 and Figure 3 do not show a notable difference.

Given that the low-detection region is likely to be coincident with the initial outbreak of the pandemic, the heterogeneous implications of the detection is also consistent with the motivational evidence presented in Section 2 (Table 2). Due to the presence of inequality, detection alone at relative low level is not effective to combat the pandemic crisis. In the next Section, we consider a complementary arrangement which could mitigate the sided effects of low detection and deliver monotonic beneficial effect of detection as suggested by column (i) and (iii) in Table 2.

5.3 Roles of social protection

Figure 8 Consumption loss and detection: with social protection



Note: This figure shows relationships between 1-year averaged consumption loss and detection for aggregate case and poor people.

Section 5.2 establishes nonlinear impacts of detection for the poor due to their financial vulnerability. In this subsection, an extended case that quarantined people are protected by the social security system is considered. Even if they cannot work after detection, they can obtain government transfers which are used for consumption. We assume that the transfer amount is equal to the income of recovered people (Eichenbaum et al. 2022). In this case, $c_t^{id,p} = c_t^{r,p}$, and $c_t^{id,w} = c_t^{r,p} + \pi_t^f$. Such an extended case is also consistent with income support programs implemented in many countries.[9]

We highlight relationships between the detection rate and consumption loss for the aggregate case and poor people in Figure 8. There are 2 important differences after accounting for the social protection for detected people. First, the relationships are likely to be monotonic and negative with social protection, implying that the livelihood for the poor under quarantine would no longer be a major threat. Second, the presence of social protection also dampens the magnitude of the recession given others as constant. For

[9]For example, the UK implemented a COVID-19 job retention scheme or furlough scheme in 2020. The scheme is a type of wage subsidy program aiming to support employees who are on furlough to receive some grants. The government is the major payer for this scheme.

instance, at 20% of detection rate, the 1-year aggregate consumption loss is 0.3% in Figure 8 while that loss is 4% in the absence of the social protection (see Figure 7). Finally, our finding is consistent with literature showing that government interventions could reduce inequality (Stantcheva 2022).

5.4 Lock-down v.s. testing

During the pandemic crisis, many countries implemented containment policies such as lock-downs to prevent the transmission of the Covid-19. In this section, we compare the effects of the lock-down with testing. In particular, we compare the evolution of the epidemic in three cases: (1) a lock-down as described in Eichenbaum et al. (2021) without detection, (2) relatively low detection rate (5%) without lock-down, and (3) relatively high detection rate (20%) without lock-down.

With the containment policy, the budget constraint for a type-i,j person becomes

$$(1-\mu_t)c_t^{i,j} = w_t n_t^{i,j} + \mathbb{1}\pi_t^f + \Gamma_t \tag{39}$$

where μ_t captures the containment rate, modelled as a tax on consumption, analogous to Farhi & Werning (2014). The proceeds due to the containment are rebated lump sum to all agents Γ_t.

Figure 9 Impacts on consumption and hours: lock-down v.s. testing

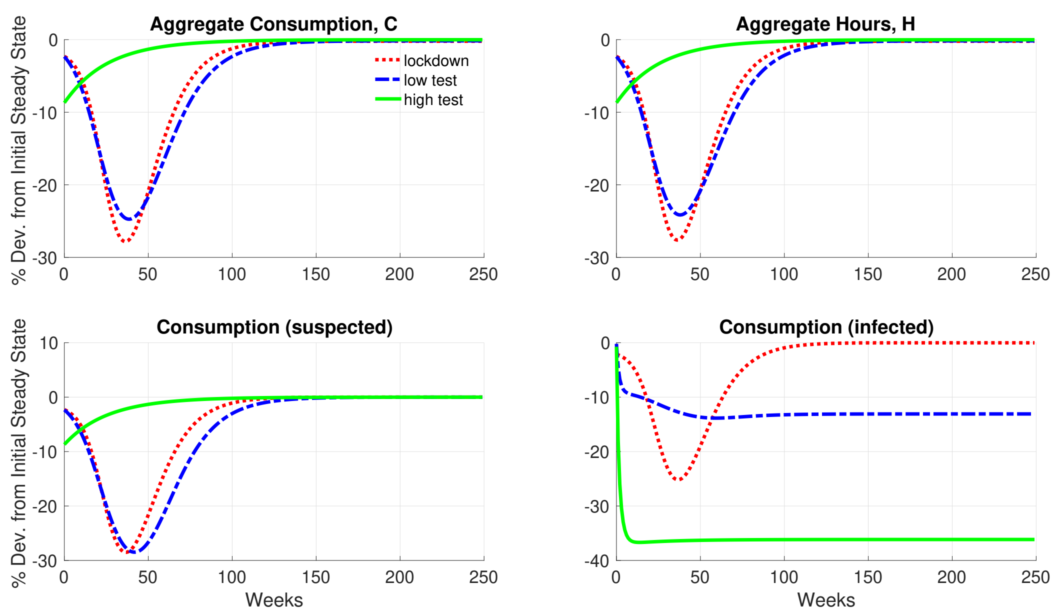
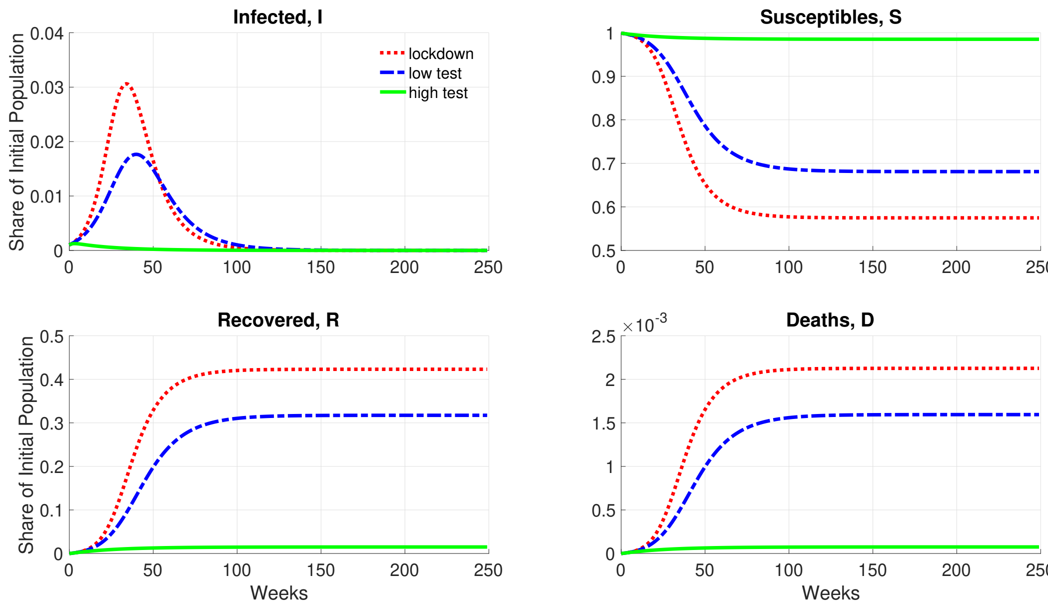


Figure 9 shows the evolution of consumption and hours at the aggregate level, and consumption for suspected and infected people. With 20% of detection rate, the economy is strongly hit by the pandemic crisis at the beginning but recover quickly. If the detection rate becomes 5%, the recession is more significant

Figure 10   The evolution of the epidemic: lock-down v.s. testing



and the recovery becomes more sluggish, despite a smaller initial response. In another case with lock-down but no detection, the evolution of the aggregate economy is similar to the low-test case.

In terms of the health outcomes, Figure 10 shows the evolution of people in different healthy categories. Not surprisingly, the relatively high detection rate leads to the least infection and death. Comparing the low-test case with the lock-down, we find that the testing, even at the relatively low level, could lead to fewer people being infected and dead. Moreover, Figure 9 and 10 together suggest an interesting finding. Between testing and lock-down leading to the similar aggregate economic performance in the pandemic crisis, the case with testing could be more effective in containment of the virus transmission, thereby leading to better health outcomes. We therefore interpret the testing case as smart quarantining with specific targets while the lock-down as massive quarantine. In this sense, the former measure is not surprisingly seen to be a more efficient tool to fight the pandemic crisis.

5.5 An attempt to relax no-reinfection assumption

In the model, we assume that recovered people have sufficient immunity so that they would not be affected again. If the mass majority of people obtains immunity, either through vaccination or recovery after infection, the spread of the pandemic would be unlikely, implying that herd immunity occurs. However, it remains questionable if the no-reinfection assumption holds. Medical research finds that the antibody of SARS-CoV-2 starts to decrease within 2–3 months after infection (e.g., Long et al. (2020)). The duration of the immunity might be shorter than other SARS-CoV or MERS-CoV. Furthermore, some recovered people got infection again though this probability is low. Moreover, we observed frequent mutations of SARS-CoV-2, such as the Delta and the Omicron variant. All these facts and findings call for investigation of implications of the pandemic when reinfection is possible. Hence, we relax the no-reinfection assumption in this subsection. By doing so, we attempt to analyze the implication of the pandemic crisis for the recession and subsequent recovery in this extended case.

Taking into account the immunity lost, the evolution of susceptible and recovered people become as follows

$$S_{t+1} = S_t - T_t + \pi^s R_t \tag{40}$$

$$R_{t+1} = R_t(1 - \pi^s) + \pi^r I_t \tag{41}$$

where π^s denotes the immunity loss rate. Equation (40) and (41) suggest that each period a fraction of recovered people becomes susceptible. The presence of the immunity lost will also change lifetime utility for recovered people.

Recovered (R)

$$\max_{c_t^r, n_t^r} U_t^r = (lnc_t^r - \frac{\theta}{2} n_t^r) + \beta[(1 - \pi^s)U_{t+1}^r + \pi^s U_{t+1}^s]$$

To begin with, we set π^s at 5%, implying that on average recovered people may significantly lose antibodies about 5 months after recovery. Note that this ratio may not be a rigorous value and it is mainly served as illustration purpose.

Figure 11 Impacts on consumption and hours: with reinfection

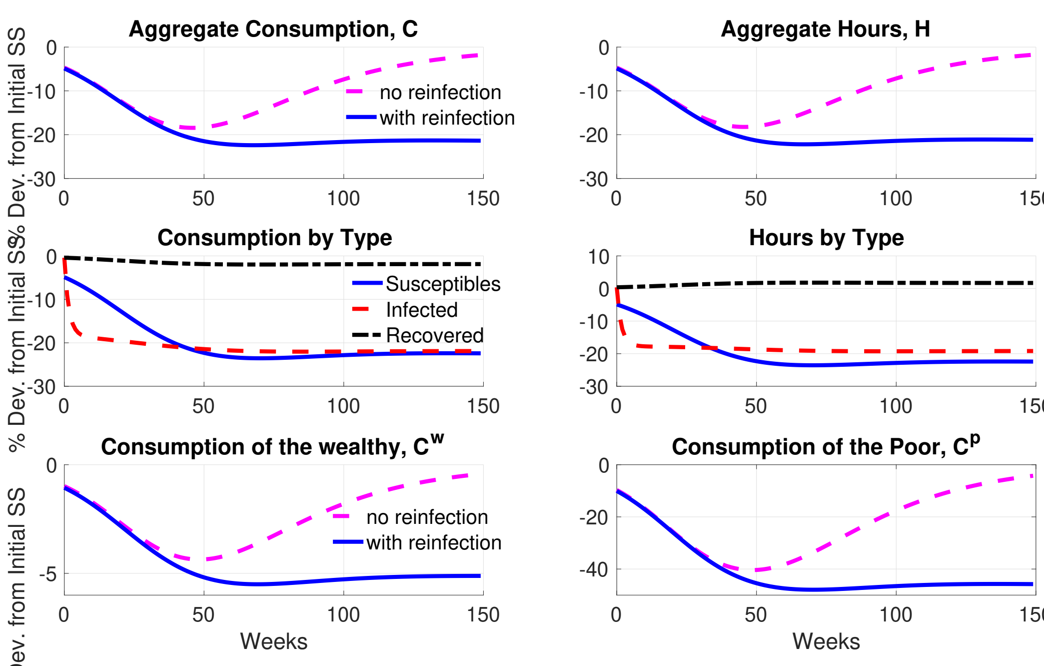
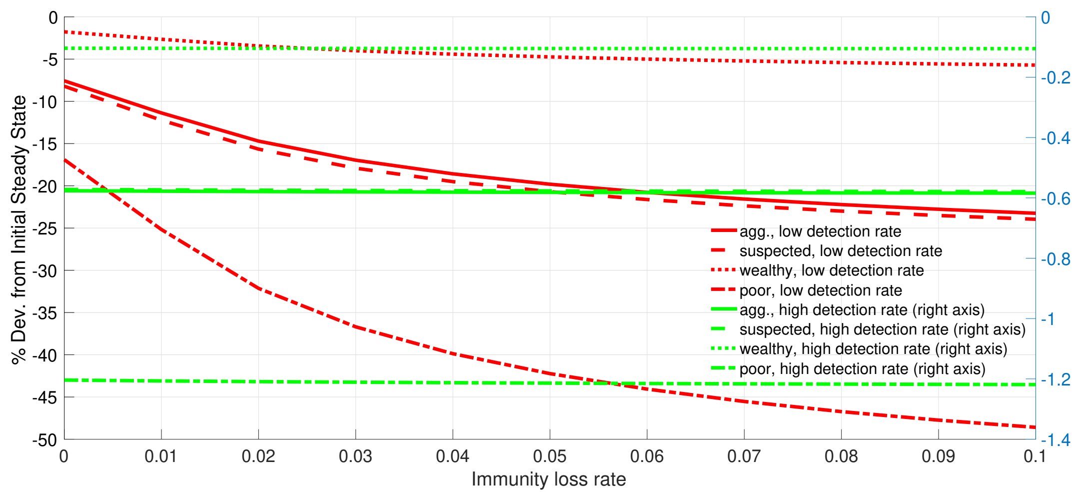


Figure 11 shows the evolution of the economy in the extended case. It shows that the pandemic could permanently affect the economy, leading to irreversible damage – a L-shaped recovery. For example, the aggregate consumption would be about 20% less than the pre-crisis level for one year after the pandemic outbreak, contrast to the benchmark case where consumption starts to recover to the pre-crisis level at that time. The lower panels of Figure 11 further shows that the impacts on households with different wealth levels also differ; the poor household is more affected. In spite of this difference, both types of households would suffer from permanent loss of consumption. The rationale is that the virus would exist with people in the long-run who have to permanently reduce consumption and working to avoid being infected.

Figure 12 Consumption loss and immunity loss rate

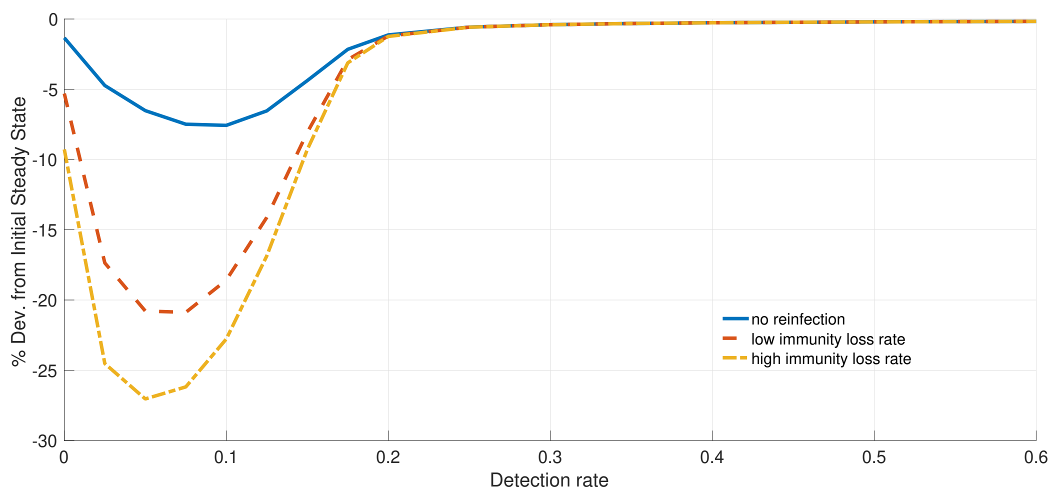


Note: This figure shows relationships between 4-year averaged consumption loss and immunity loss rate for aggregate case, suspected people, wealthy people and poor people. For each case, we further classify it by a relatively low detection rate (10%, red lines) and a relatively high detection rate (20%, green lines).

We further investigate relationships between the magnitude of the consumption loss and immunity loss rate, as depicted in Figure 12. Since the reinfection is more likely to affect long-run dynamics of the pandemic recession as shown in Figure 11, we show 4-year averaged consumption loss rather than the 1-year loss in Figure 12. In general, this figure shows positive relationships between the speed of losing the immunity and magnitude of consumption loss. Comparatively, the relationships are much steeper when the detection rate is relatively low. In particular, the most pronounced impacts are found from the poor, indicating that they are more likely to suffer from the immunity loss issue than the rich. The decision of economic activities for the poor could be the most sensitive to the strength of antibody. On the contrary, the relationships become insensitive when the detection rate is relatively high. Therefore, the presence of reinfection might exacerbate inequality in the pandemic recession given the relatively low detection rate. However, accurate and extensive testing could be helpful to deal with the reinfection issue, to prevent deep recessions and enlarged inequality.

Finally, we investigate how the testing may interact with the immunity loss issue. Figure 13 shows that

Figure 13 Consumption loss and detection: with reinfection



Note: This figure shows relationships between 4-year averaged aggregate consumption loss and detection rate. We include 3 cases in the figure: no-reinfection (blue), a relatively low immunity loss rate (5%), and a relatively high immunity loss rate (10%).

high detection rates could significantly mitigate the adverse effects due to the reinfection. For example, the gap between the red line and the blue line in Figure 13 becomes negligible with high detection rates. This finding implies a complementary role of detection to the vaccine in rescue. Even if effects of the vaccine might not be long-lasting or weakened, e.g., due to potential mutation of the virus, efficient and swift tests could be useful.

6 Conclusion

The COVID-19 pandemic raised challenges for the economics researchers to address both the economic and health consequences of the crisis, resulting in the publication of studies addressing the interaction between the epidemic and the economy. This paper further that literature by addressing an additional set of important implications of the pandemic crisis, and shedding light on the recession and recovery of the crisis. To achieve this, we develop a SIR-macro model with virus detection and income inequality for households. Essentially, we find a two-way relationship between the pandemic recession and inequality, both of which can exacerbate one another. We show that such a vicious circle could be broken by accurate and extensive testing. In order to maximize the benefits of the virus detection, especially for the poor, some complementary arrangements such as social protection should be provided. These policies are important for the containment of the virus in the early outbreak of the pandemic when testing capacity and accuracy were low.

Our framework provides important insights based on a simple model, highlighting several fundamental forces of the pandemic crisis. Further research could therefore enhance our framework by incorporating some important real-world factors such as considering the role of monetary and fiscal policies in the dynamic of inequality during the pandemic recession. Moreover, it is important to consider sector heterogeneity and study the supply-sided implications to further identify the long-run effects of the COVID-19 pandemic.

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Appendix A Equilibrium Conditions

$$T_t = \pi_1(S_t C_t^s)(I_t^u C_t^{iu}) + \pi_2(S_t N_t^s)(I_t^u N_t^{iu}) + \pi_3 S_t I_t^u \tag{A1}$$

$$S_{t+1} = S_t - T_t \tag{A2}$$

$$I_{t+1}^u = I_t^u + T_t - (\pi_r + \pi_d + \pi_u) I_t^u \tag{A3}$$

$$I_{t+1}^d = I_t^d + \pi_u I_t^u - (\pi_r + \pi_d) I_t^d \tag{A4}$$

$$I_t = I_t^d + I_t^u \tag{A5}$$

$$R_{t+1} = R_t - \pi_r I_t \tag{A6}$$

$$D_{t+1} = D_t + \pi_d I_t \tag{A7}$$

$$Pop_{t+1} = Pop_t + \pi_d I_t \tag{A8}$$

$$U_t^{s,p} = u(c_t^{s,p}, n_t^{s,p}) + \beta[(1-\tau_t)U_{t+1}^{s,p} + \tau_t U_{t+1}^{iu,p}] \tag{A9}$$

$$U_t^{s,w} = u(c_t^{s,w}, n_t^{s,w}) + \beta[(1-\tau_t)U_{t+1}^{s,w} + \tau_t U_{t+1}^{iu,w}] \tag{A10}$$

$$U_t^{iu,p} = u(c_t^{iu,p}, n_t^{iu,p}) + \beta[(1-\pi_u-\pi_r-\pi_d)U_{t+1}^{iu,p} + \pi_u U_{t+1}^{id,p} + \pi_r U_{t+1}^{r,p}] \tag{A11}$$

$$U_t^{iu,w} = u(c_t^{iu,w}, n_t^{iu,w}) + \beta[(1-\pi_u-\pi_r-\pi_d)U_{t+1}^{iu,w} + \pi_u U_{t+1}^{id,w} + \pi_r U_{t+1}^{r,w}] \tag{A12}$$

$$U_t^{id,p} = u(c_t^{id,p}, n_t^{id,p}) + \beta[(1-\pi_r-\pi_d)U_{t+1}^{id,p} + \pi_r U_{t+1}^{r,p}] \tag{A13}$$

$$U_t^{id,w} = u(c_t^{id,w}, n_t^{id,w}) + \beta[(1-\pi_r-\pi_d)U_{t+1}^{id,w} + \pi_r U_{t+1}^{r,w}] \tag{A14}$$

$$U_t^{r,p} = u(c_t^{r,p}, n_t^{r,p}) + \beta U_{t+1}^{r,p} \tag{A15}$$

$$U_t^{r,w} = u(c_t^{r,w}, n_t^{r,w}) + \beta U_{t+1}^{r,w} \tag{A16}$$

$$\frac{1}{c_t^{s,p}} = \lambda_t^{s,p} + \beta\pi_1 I_t^u C_t^{iu}(U_{t+1}^{s,p} - U_{t+1}^{iu,p}) \tag{A17}$$

$$\frac{1}{c_t^{s,w}} = \lambda_t^{s,w} + \beta\pi_1 I_t^u C_t^{iu}(U_{t+1}^{s,w} - U_{t+1}^{iu,w}) \tag{A18}$$

$$\theta n_t^{s,p} = \lambda_t^{s,p} w_t - \beta\pi_2 I_t^u N_t^{iu}(U_{t+1}^{s,p} - U_{t+1}^{iu,p}) \tag{A19}$$

$$\theta n_t^{s,w} = \lambda_t^{s,w} A\Theta_t - \beta\pi_2 I_t^u N_t^{iu}(U_{t+1}^{s,w} - U_{t+1}^{iu,w}) \tag{A20}$$

$$\theta n_t^{iu,p} = \frac{w_t}{c_t^{iu,p}} \tag{A21}$$

$$\theta n_t^{iu,w} = \frac{A\Theta_t}{c_t^{iu,w}} \tag{A22}$$

$$n_t^{id,p} = 0 \tag{A23}$$

$$n_t^{id,w} = 0 \tag{A24}$$

$$c_t^{id,p} = 0 \tag{A25}$$

$$c_t^{id,w} = \frac{\gamma - 1}{\gamma} A N_t \Theta_t \tag{A26}$$

$$\theta n_t^{r,p} = \frac{w_t}{c_t^{r,p}} \tag{A27}$$

$$\theta n_t^{r,w} = \frac{A_t \Theta_t}{c_t^{r,w}} \tag{A28}$$

$$S_t C_t^s + IU_t C_t^{iu} + ID_t C_t^{id} + R_t C_t^r = A N_t \tag{A29}$$

$$S_t N_t^s + IU_t N_t^{iu} + R_t N_t^r = N_t \tag{A30}$$

$$C_t^i = \chi c_t^{i,w} + (1-\chi) c_t^{i,r}, \qquad i = s, iu, id, r \tag{A31}$$

$$N_t^i = \chi n_t^{i,w} + (1-\chi) n_t^{i,r}, \qquad i = s, iu, id, r \tag{A32}$$

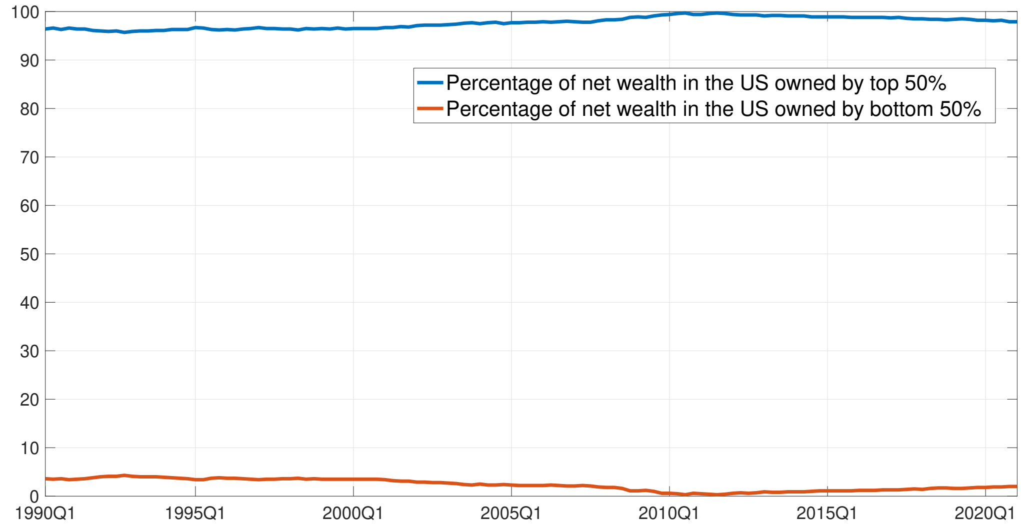
Appendix B Data

Table 4 Data used in the empirical analysis

Variables	Description	Source
Growth	GDP Growth	WDI, OECD
	GDP per capita Growth	WDI
Gini	Gini index	WDI, SWIID
Test	Weekly testing rate per 100000 people	ECDC
pop	Population, total	WDI
cpi	Consumer price index (2010 = 100)	WDI, OECD
gov	General government final consumption expenditure (% of GDP)	WDI, OECD
con	Households and NPISHs final consumption expenditure (% of GDP)	WDI, OECD
inv	Gross capital formation (% of GDP)	WDI, OECD
health_exp	Domestic general government health expenditure (% of GDP)	WDI
employ	Employment to population ratio, 15+, total (%) (modeled ILO estimate)	WDI

Note: WDI represents World Development Indicators, OECD represents OECD quarterly national account database, SWIID represents the Standardized World Income Inequality Database, and ECDC represents European Centre for Disease Prevention and Control COVID-19 datasets. Yearly data of controls are obtained from WDI, while quarterly data are from OECD. The missing value of Gini is interpolated according to the previous data and other development Indicators. Test data are aggregated from weekly to quarterly frequency.

Figure 14 Net wealth in the US: comparing the top and bottom 50%



Source: the Distributional Financial Accounts, https://www.federalreserve.gov/releases/z1/dataviz/dfa/index.html

Figure 15 Testing and growth